

Wednesday Nov. 28
Lecture 23

Is an array sorted?

```
int[] a1 = {};  
print(isSorted(a1))
```

```
boolean isSorted(int[] a) {  
→ return isSortedHelper(a, 0, a.length - 1);  
}
```

```
boolean isSortedHelper(int[] a, int from, int to) {
```

```
if (from > to) { /* base case 1: empty range */  
→ return true;
```

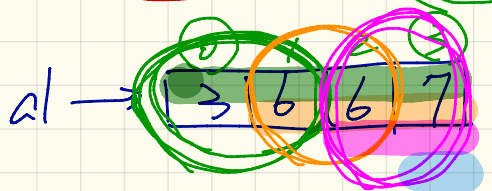
```
else if (from == to) { /* base case 2: range of one element */  
→ return true;
```

```
else {  
→ return a[from] <= a[from + 1]  
    && isSortedHelper(a, from + 1, to);  
}
```

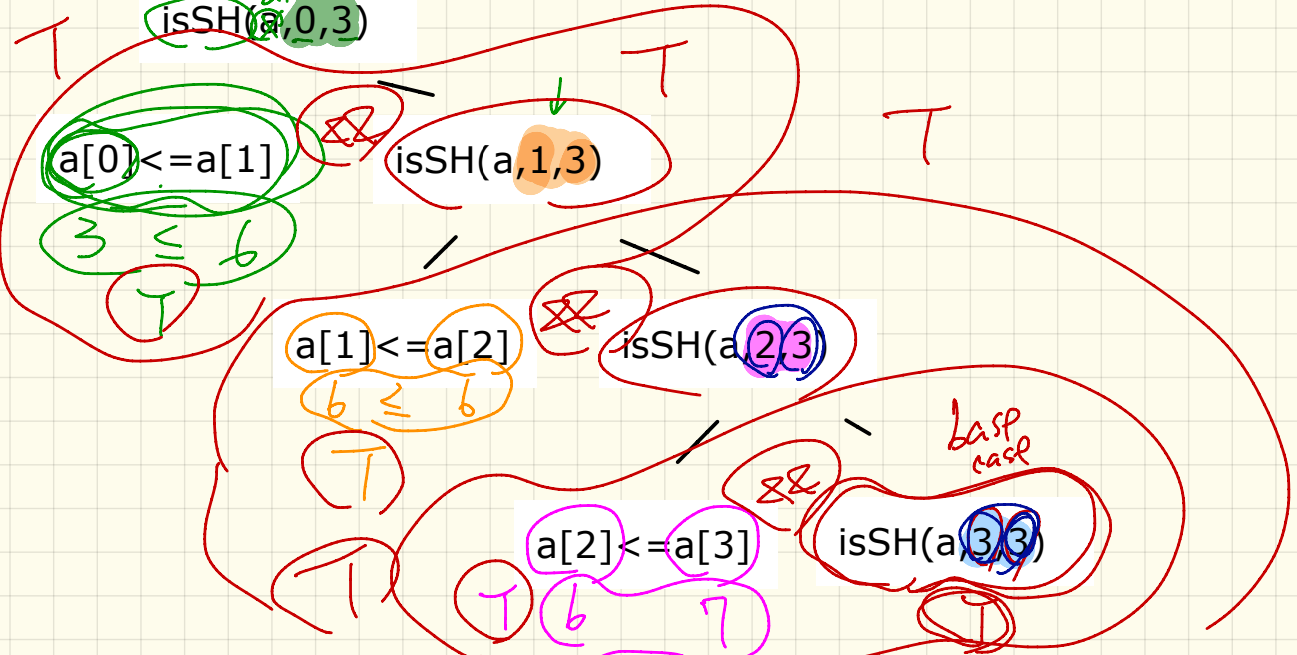
Tracing isSorted

Say $a1 = \{3, 6, 6, 7\}$, $a2 = \{3, 6, 5, 7\}$

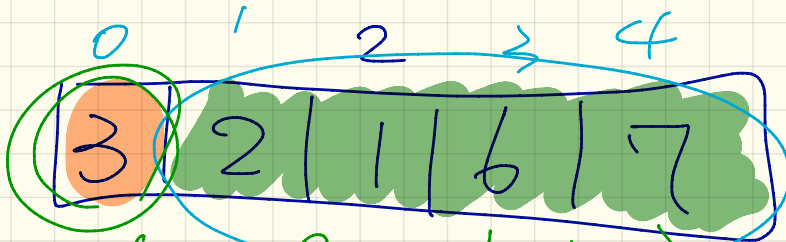
$a1$
 $isSorted(a)$



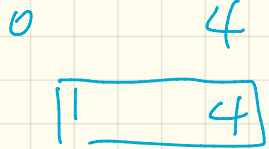
$isSH(a, 0, 3)$



1



$\text{min}(a, 0, a.\text{length}-1)$

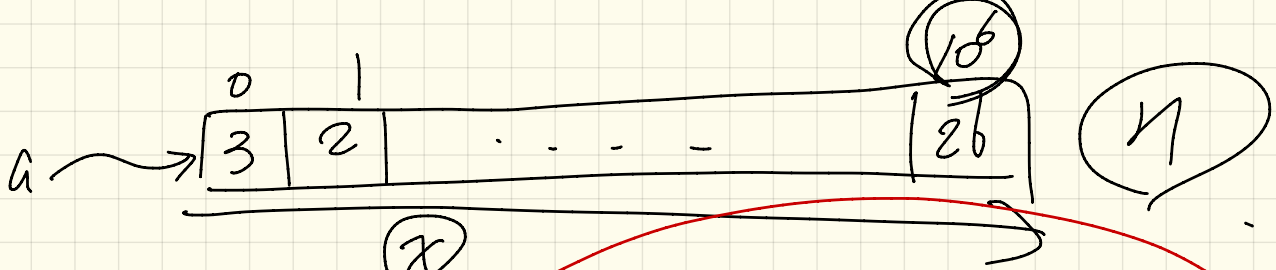


$\text{if}(a \text{ is empty}) \{ \text{no min} \}$

$\text{else if}(a \text{ is size } 1) \{ \text{return } a[0] \}$

$\text{else} \{$

$\text{int minOfRest} = \text{min}(a, \text{from} + 1, \text{to});$
 $\text{if}(a[0] < \text{minOfRest}) \{ \text{return } a[0]; \}$
 $\text{else} \{ \text{return } \text{minOfRest}; \}$



Search (23)

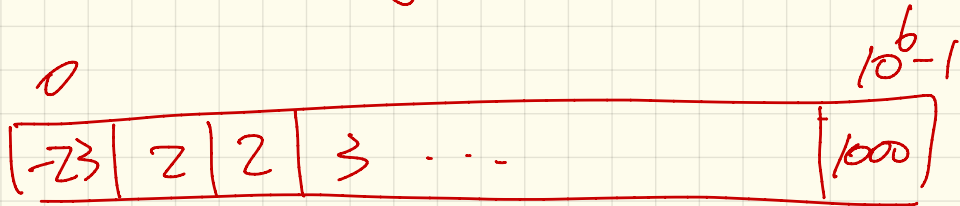
```
for (int i = 0; i < a.length; i++) {
    if (a[i] == 23) { return true; }
}
```

$O(n)$

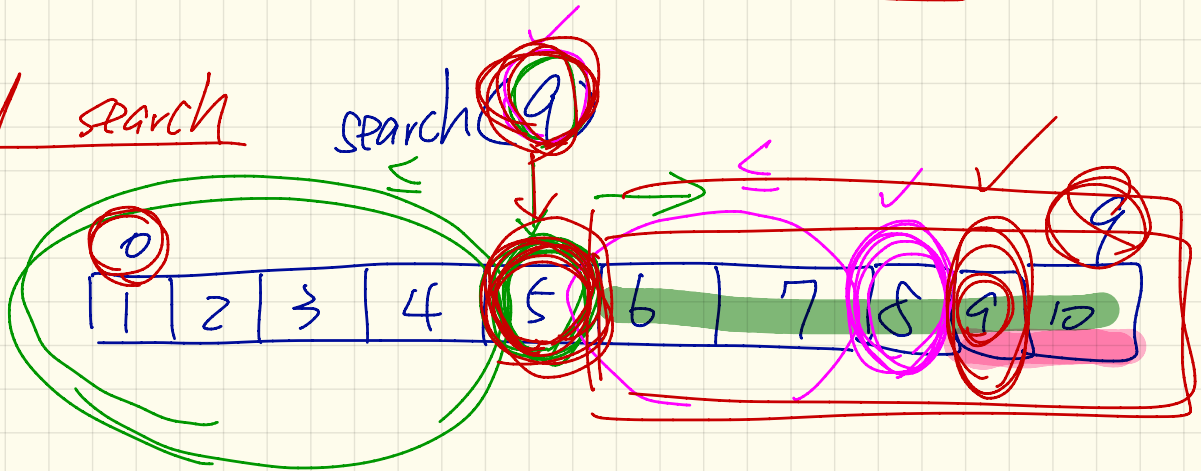
10^6

return false;

Assume input array is sorted



Binary search

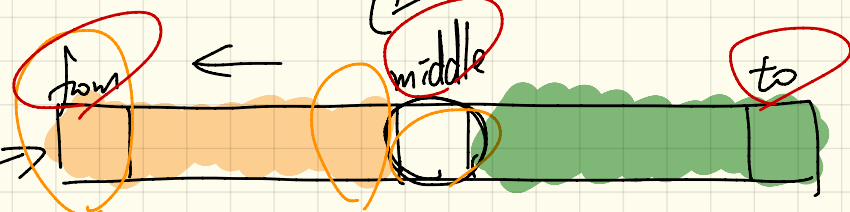


Binary Search

```
boolean binarySearch(int[] sorted, int key) {  
    return binarySearchHelper(sorted, 0, sorted.length - 1, key);  
}  
  
boolean binarySearchHelper(int[] sorted, int from, int to, int key)  
if (from > to) { /* base case 1: empty range */  
    return false; }  
else if (from == to) { /* base case 2: range of one element */  
    return sorted[from] == key; }  
else {  
    int middle = (from + to) / 2;  
    int middleValue = sorted[middle];  
    → if (key < middleValue) {  
        return binarySearchHelper(sorted, from, middle - 1, key);  
    }  
    → else if (key > middleValue) {  
        return binarySearchHelper(sorted, middle + 1, to, key);  
    }  
    else { return true; }  
}
```

in ascending order!

sorted
binSearch (arr , from , to , key)



if (key < sorted [middle]) {

binSearch (sorted , from , middle - 1 , key);

} else if (key > sorted [middle]) {

binSearch (sorted , middle + 1 , to , key);

Exercise

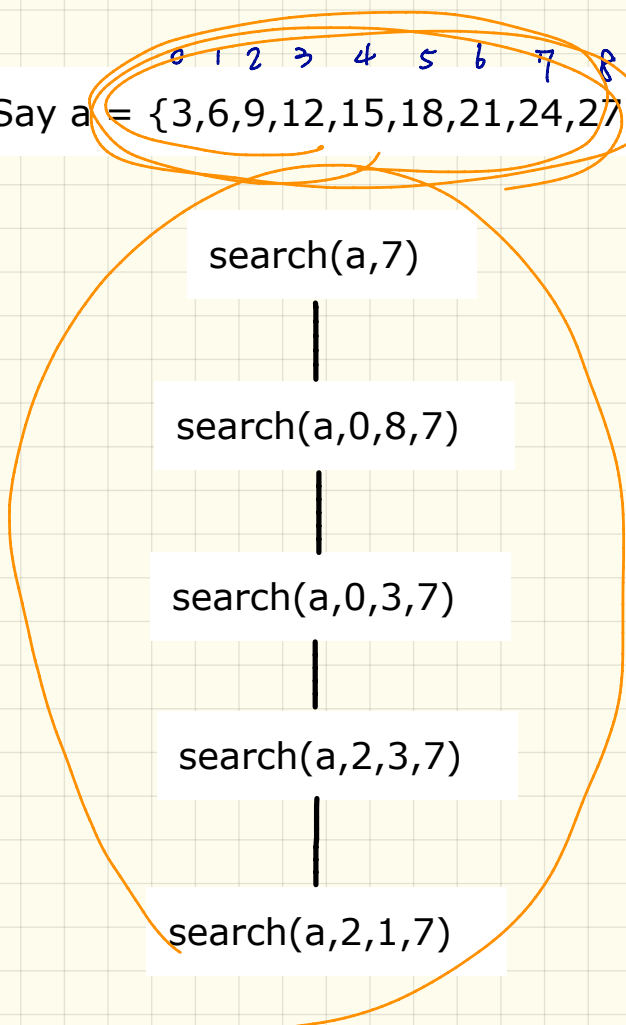
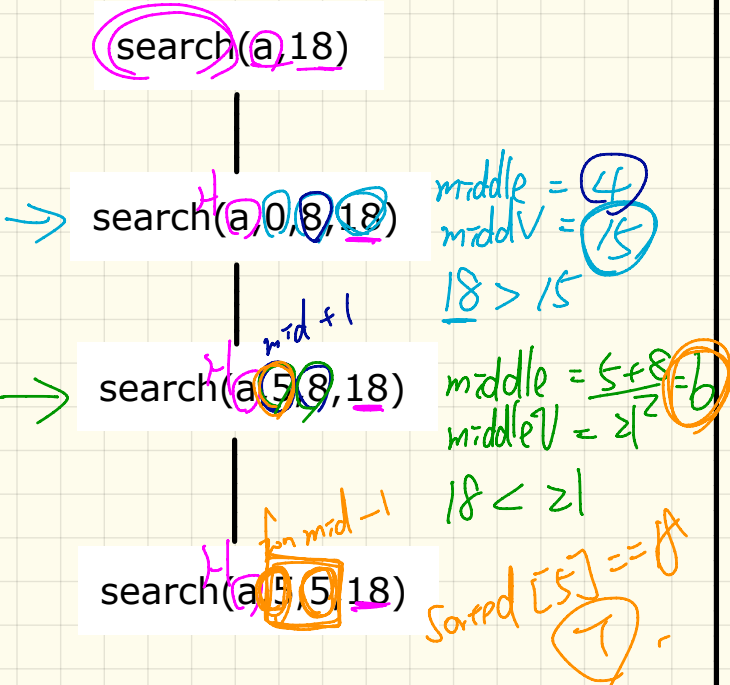
Modify the BinS.
so that the input
array is sorted
in descending
order

Binary Search: Tracing

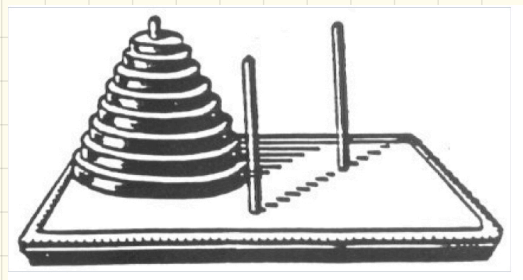
0 1 2 3 4 5 6 7 8
 m
 n

Say $a = \{3, 6, 9, 12, 15, 18, 21, 24, 27\}$

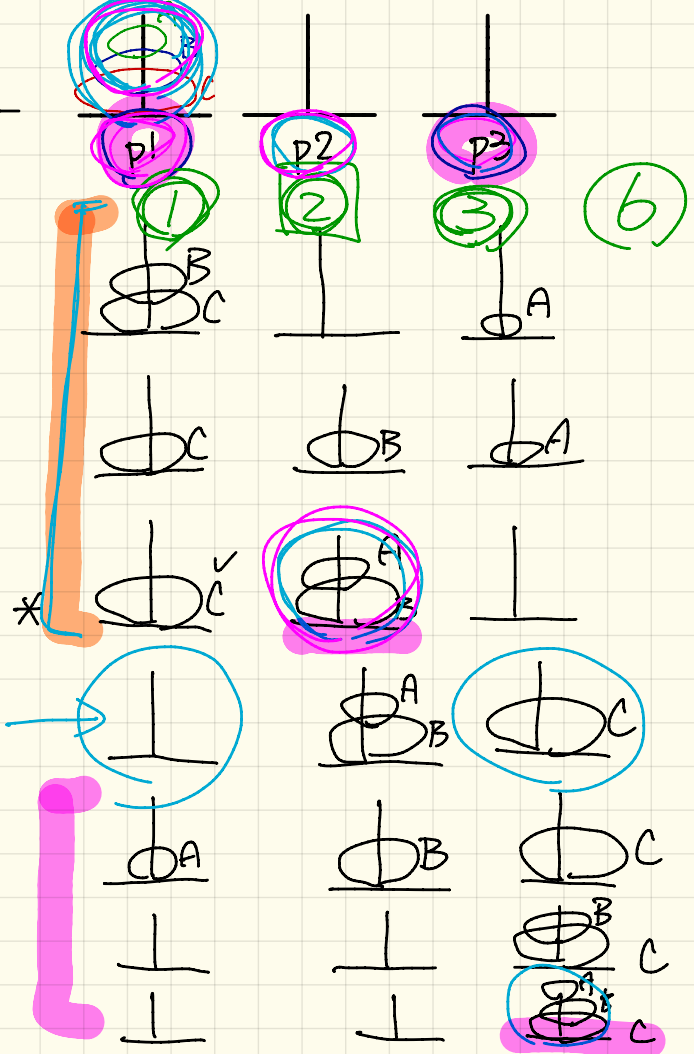
Say $a = \{3, 6, 9, 12, 15, 18, 21, 24, 27\}$



Tower of Hanoi: Strategy



Consider 3 disks $A < B < C$



Tower of Hanoi : Java

```
void towerOfHanoi(String[] disks) {  
    tohHelper (disks, 0, disks.length - 1, 1, 3);  
}  
void tohHelper(String[] disks, int from, int to, int p1, int p2) {  
    if (from > to) { }  
    else if (from == to) {  
        print("move " + disks[to] + " from " + p1 + " to " + p2);  
    }  
    else {  
        int intermediate = 6 - p1 - p2;  
        tohHelper (disks, from, to - 1, p1, intermediate);  
        print("move " + disks[to] + " from " + p1 + " to " + p2);  
        tohHelper (disks, from, to - 1, intermediate, p2);  
    }  
}
```

Handwritten annotations:

- $A \rightarrow B \rightarrow C$ (circled in pink)
- Arrow pointing from $p1$ to $p3$ with text "move from $p1$ to $p3$ "
- Arrow pointing from $p1$ to $p2$ with text "move from $p1$ to intermediate $(p2)$ "
- Various parameters and expressions in the code are circled in pink.

Say disks = {A,B,C}.

Consider towerOfHoni(disks) which calls:
tohHelper(disks, 0, disks.length - 1, 1, 3)